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Alfvén solitons in the coupled derivative nonlinear Schrödinger system with symbolic computation

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Abstract
The propagation of nonlinear Alfvén waves in magnetized plasmas with right and left circular polarizations is governed by the coupled derivative nonlinear Schrödinger (CDNLS) system. The integrability of this system is indicated by the existence of two gauge-equivalent Lax pairs and infinitely many independent conservation laws. With symbolic computation, the analytic one- and two-soliton solutions are obtained via the Hirota bilinear method. The propagation characteristics of the Alfvén waves are discussed through qualitative analysis. The collision dynamics of the CDNLS solitons is found to be characterized by the invariance of the soliton velocities and widths, parameter-dependent changes of the soliton amplitudes and conservation of the total energy of right- and left-polarized components. The parametric condition for the amplitude-preserving collision occurring in each component is explicitly given.

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1. Introduction

Plasmas can be found everywhere in the Universe [1]. It is important to study the dynamics of Alfvén waves, which are the most 'robust' plasma oscillations in a magnetized system and widespread in space and laboratory plasmas [2, 3]. Numerous observations suggest that Alfvén wave trains and Alfvénic turbulence widely exist in the Earth's magnetosphere [4], solar winds [5], planetary bow shocks [6], dusty cometary tails [7], interplanetary shocks

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[8] and other cosmic environments. References [9, 10] have also reported the experimental observation of dispersive Alfvén waves in laboratory plasmas, e.g. in tokamaks and linear devices.

Actual observations have shown that the nonlinearity as well as the dispersion in a plasma plays a non-negligible role in the propagation of Alfvén waves [4–10]. In a particular case when the effects of nonlinear steepening and dispersive broadening balance each other, Alfvén waves evolve into solitons which exhibit stationary shapes over a long time [11]. The derivative nonlinear Schrödinger (DNLS) equation is known to be a model governing the dynamical behavior of weakly nonlinear and weakly dispersive Alfvén waves in the one-dimensional approximation [12–18]. The DNLS models are usually derived from the two-fluid equations and the generalized Ohm's law together with the adiabatic equation of state under the following assumptions [11–20].

i. With the ignorance of the variation of density and temperature, and the charge separation between electrons and ions, the plasma can be reasonably assumed to be uniform, adiabatic and quasi-neutral. The plane magnetohydrodynamic waves are restricted to propagate in the x direction (the x component of the magnetic wave field $\mathbf{B}$ is a constant) along a uniform, stationary background magnetic field $\mathbf{B}_0$, which lies in the x–y plane inclined at an angle θ with respect to the x-axis.

ii. Since the phase velocity of Alfvén waves is much smaller than the speed of the light c, the contribution from the displacement currents is naturally negligible. It also requires that the pressure tensors for electrons and ions are diagonal and isotropic, and the compression and expansion occur isentropically.

iii. Long wavelength

$$\frac{d_i}{L} = O(\epsilon), \quad \frac{r_i}{L} = O(\epsilon), \quad \epsilon \ll 1,$$

where $d_i = V_A/\Omega_i$ is the ion inertial length, $V_A = B_0/\sqrt{4\pi\,\rho_0}$ (B_0 is the static magnetic field strength and ρ_0 is the ambient mass density) is the Alfvén velocity, $\Omega_i = Z_i B_0/cm_i$ (Z_i is the ion charge and m_i is the ion mass) is the ion cyclotron frequency, r_i is the ion gyro radius, L is the characteristic length and ϵ is a small parameter characterizing the weak nonlinearity and weak dispersion.

iv. Small amplitude

$$\frac{|\tilde{B}|^2}{B_0^2} = O(\epsilon),$$

where $\tilde{B} = \epsilon^{-1/2}(\tilde{B}_y + \mathrm{i}\,\tilde{B}_z)$ with $\tilde{B}_y$ and $\tilde{B}_z$ as two components of the magnetic wave field in the y and z directions orthogonal to x. The DNLS equation is invalid for very large amplitude waves because those terms higher than cubic nonlinearities are not included in the derivation.

v. Slow evolution

$$\frac{1}{\Omega_i}\frac{\partial}{\partial_t} = O(\epsilon^2),$$

which implies the constraint that no disturbance is present which propagates at any other of the characteristic velocity.

The DNLS equation was initially derived from the Vlasov kinetic equation [12], and later it was obtained on the basis of Hall magnetohydrodynamics for cold plasmas [13, 14]. In warm two-fluid plasmas, the DNLS model has the following form [15–18]:

$$\mathrm{i}\,\tilde{B}_{\tilde{t}} - \mathrm{i}\frac{V_A}{\beta' B_0^2}\left[\tilde{B}\left(|\tilde{B}|^2 - B_{0y}^2\right)\right]_{\tilde{x}} + \frac{V_A^2}{2\Omega_i}\,\tilde{B}_{\tilde{x}\tilde{x}} = 0, \tag{1}$$

where $\tilde{x} = \epsilon\,(x - V_A t)$ is the space coordinate in a frame of reference moving with the Alfvén velocity, $\tilde{t} = \epsilon^2 t$ is the stretched time coordinate, $\beta' = 4(\beta - 1)$, $\beta = c_s^2/V_A^2$ (c_s is the ion sound velocity) represents the ratio of the kinetic pressure to magnetic pressure and $B_{0y} = B_0 \sin\theta$ is a constant. Equation (1) is relevant for describing the nonlinear evolution of Alfvén waves either exactly parallel ($B_{0y} = 0$) or at a small angle ($B_{0y} \neq 0$) to the background magnetic field. The complex transverse magnetic field $\tilde{B}$ and its complex conjugate $\tilde{B}^*$ signify the right and left circularly polarized Alfvén waves, respectively. One should bear in mind that equation (1) is invalid for $\beta \simeq 1$ because a 'static' approximation used in the derivation is not appropriate [15, 16]. In [21], a new set of coupled equations has been derived to describe finite amplitude, dispersive, circularly polarized Alfvén waves in the range $\beta \simeq 1$.

An advantage of the DNLS equation for the nonlinear Alfvén wave description lies in that it is a completely integrable model which can be solved by the inverse scattering transform (IST) [22]. In the past three decades, people have detailed its various integrable properties including the Hamiltonian structures [23], multi-soliton solutions [22, 24], bilinear representation [24], an infinite number of conservation laws [22, 25], Painlevé property [26] and Darboux transformation [27]. Of current interest, the investigation on the DNLS equation can be seen from the aspect of N-soliton solutions and perturbation theory with nonvanishing boundary conditions based on the IST method [28]. In addition, physicists have also derived the inhomogeneous DNLS model [29], modified vector DNLS system [30] and some other extended DNLS equations [31], subject to more complicated plasma environments.

As shown in [15], equation (1) can be reduced to a simpler form

$$\mathrm{i}\,q_\tau + \mathrm{i}\,q_\xi - \frac{\mathrm{i}}{\beta'}\left(q\,|q|^2\right)_\xi + \mu\,q_{\xi\xi} = 0 \quad \text{with} \quad \mu = \frac{V_A^2}{2\Omega_i}, \tag{2}$$

by the transformations

$$q(\xi,\tau) = \frac{\sqrt{|\beta'|}\,\tilde{B}(\tilde{x},\tilde{t})}{B_{0y}}, \qquad \xi = \frac{B_{0y}^2 V_A \tilde{x}}{\beta' B_0^2}, \qquad \tau = \frac{B_{0y}^4 V_A^2 \tilde{t}}{\beta'^2 B_0^4}. \tag{3}$$

Although a number of previous studies were concerned with the application of equation (2) to the single circularly polarized Alfvén wave, there has not been much attention paid to arbitrary circularly polarized Alfvén waves which are in nature closer to realistic situations. In this case, we consider that q in equation (2) is a jointly coupled field and comprised the right and left circularly polarized Alfvén waves [32]. Hence, we can derive the coupled DNLS (CDNLS) system in the form

$$\mathrm{i}\,q_{1,\tau} + \mathrm{i}\,q_{1,\xi} - \frac{\mathrm{i}}{\beta'}\left[q_1\left(|q_1|^2 + |q_2|^2\right)\right]_\xi + \mu q_{1,\xi\xi} = 0, \tag{4a}$$

$$\mathrm{i}\,q_{2,\tau} + \mathrm{i}\,q_{2,\xi} - \frac{\mathrm{i}}{\beta'}\left[q_2\left(|q_2|^2 + |q_1|^2\right)\right]_\xi + \mu q_{2,\xi\xi} = 0, \tag{4b}$$

where q_1 and q_2 represent the right- and left-polarized Alfvén wave components, respectively.

Generally, the integrability of a given nonlinear system is the foremost concern in soliton theory. As investigated in [32], system (4) is known to be integrable in the sense that it can be associated with a Lax pair in the 3×3 Kaup–Newell (KN) scheme [22]. To collect more compelling evidence for the integrability of system (4), section 2 will show that this system can be expressed as the compatibility condition of another Lax pair which is gauge equivalent to the 3×3 KN system, and section 3 will further prove that it also possesses an infinite number of independent conservation laws. We note that in recent studies attention has been paid to the soliton dynamics of the coupled nonlinear Schrödinger equations because the multi-component structure possibly makes the occurrence of some interaction features such as the intensity redistribution among colliding solitons in all the components, amplitude-dependent phase shifts and changes in relative separation distances [33–35]. Therefore, a natural question arises: does the derivative cubic nonlinearity coupling lead some unusual Alfvén soliton dynamical behaviors which are different from those occurring in the single DNLS equation? From this consideration, section 4 will be devoted to making clear the dynamics of Alfvén solitons with right- and left-polarized components. Finally, section 5 will address the conclusions of this paper. We hope that the Alfvén soliton features obtained in this work will provide a theoretical insight into various nonlinear Alfvén wave phenomena in space and astrophysical plasmas.

2. Two types of Lax pairs and gauge equivalence

The Lax pair assures the integrability of a nonlinear evolution equation (NLEE) and plays a role in solving its initial-value problem [36]. Following the procedure generalizing the 2×2 KN scheme to the 3×3 case [32, 37], we find that system (4) admits one type of Lax pair as below:

$$\Psi_\xi = U^{(I)}\Psi = \left[\lambda^2 U_0^{(I)} + \lambda U_1^{(I)}\right]\Psi, \tag{5a}$$

$$\Psi_\tau = V^{(I)}\Psi = \left[\lambda^4 V_0^{(I)} + \lambda^3 V_1^{(I)} + \lambda^2 V_2^{(I)} + \lambda V_3^{(I)}\right]\Psi, \tag{5b}$$

where $\Psi = (\psi_1, \psi_2, \psi_3)^T$ (the superscript T denotes the vector transpose) is the vector eigenfunction, λ is the spectral parameter and the matrices $U_k^{(I)}$, $V_l^{(I)}$ ($k = 0, 1$; $l = 0, 1, 2, 3$) are expressible in the form

$$U_0^{(I)} = \begin{pmatrix} -\mathrm{i} & 0 & 0 \\ 0 & \mathrm{i} & 0 \\ 0 & 0 & \mathrm{i} \end{pmatrix}, \quad U_1^{(I)} = \begin{pmatrix} 0 & q_1 & q_2 \\ \frac{1}{\beta'\mu} q_1^* & 0 & 0 \\ \frac{1}{\beta'\mu} q_2^* & 0 & 0 \end{pmatrix}, \tag{6}$$

$$V_0^{(I)} = 2\mu U_0^{(I)}, \quad V_1^{(I)} = 2\mu U_1^{(I)}, \tag{7}$$

$$V_2^{(I)} = \begin{pmatrix} \mathrm{i} - \frac{\mathrm{i}}{\beta'}\left(|q_1|^2 + |q_2|^2\right) & 0 & 0 \\ 0 & -\mathrm{i} + \frac{\mathrm{i}}{\beta'}|q_1|^2 & \frac{\mathrm{i}}{\beta'} q_2 q_1^* \\ 0 & \frac{\mathrm{i}}{\beta'} q_1 q_2^* & -\mathrm{i} + \frac{\mathrm{i}}{\beta'}|q_2|^2 \end{pmatrix}, \tag{8}$$

$$V_3^{(I)} = \begin{pmatrix} 0 & \mathrm{i}\mu q_{1,\xi} - q_1 + \frac{1}{\beta'}\Delta_1 & \mathrm{i}\mu q_{2,\xi} - q_2 + \frac{1}{\beta'}\Delta_2 \\ -\frac{\mathrm{i}}{\beta'} q_{1,\xi}^* - \frac{1}{\beta'\mu} q_1^* + \frac{1}{\beta'^2\mu}\Delta_1^* & 0 & 0 \\ -\frac{\mathrm{i}}{\beta'} q_{2,\xi}^* - \frac{1}{\beta'\mu} q_2^* + \frac{1}{\beta'^2\mu}\Delta_2^* & 0 & 0 \end{pmatrix}, \tag{9}$$

with $\Delta_1 = q_1(|q_1|^2 + |q_2|^2)$ and $\Delta_2 = q_2(|q_1|^2 + |q_2|^2)$. It is remarked that the Lax pair (5) together with equations (6)–(9) agrees with the result in [32] in the sense of a scale transformation of independent variables. In addition, based on the matrix-form inverse scattering problem proposed in [37] (i.e. system (4.1) there), we can obtain another type of Lax pair associated with system (4), as follows:

$$\Phi_\xi = U^{(II)}\Phi = \left[\lambda' U_0^{(II)} + U_1^{(II)}\right]\Phi, \tag{10a}$$

$$\Phi_\tau = V^{(II)}\Phi = \left[\lambda'^2 V_0^{(II)} + \lambda' V_1^{(II)} + V_2^{(II)}\right]\Phi, \tag{10b}$$

with

$$U_0^{(II)} = \begin{pmatrix} -\mathrm{i} & q_1 & q_2 \\ 0 & \mathrm{i} & 0 \\ 0 & 0 & \mathrm{i} \end{pmatrix}, \quad U_1^{(II)} = \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{\beta'\mu} q_1^* & 0 & 0 \\ \frac{1}{\beta'\mu} q_2^* & 0 & 0 \end{pmatrix}, \quad V_0^{(II)} = 2\,\mu U_0^{(II)}, \tag{11}$$

$$V_1^{(II)} = \begin{pmatrix} \mathrm{i} - \frac{\mathrm{i}}{\beta'}(|q_1|^2 + |q_2|^2) & \frac{1}{\beta'}\Delta_1 - q_1 + \mathrm{i}\mu q_{1,\xi} & \frac{1}{\beta'}\Delta_2 - q_2 + \mathrm{i}\mu q_{2,\xi} \\ \frac{2}{\beta'} q_1^* & -\mathrm{i} + \frac{\mathrm{i}}{\beta'}|q_1|^2 & \frac{\mathrm{i}}{\beta'} q_2 q_1^* \\ \frac{2}{\beta'} q_2^* & \frac{\mathrm{i}}{\beta'} q_1 q_2^* & -\mathrm{i} + \frac{\mathrm{i}}{\beta'}|q_2|^2 \end{pmatrix}, \tag{12}$$

$$V_2^{(II)} = \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{\beta'^2\mu}\Delta_1^* - \frac{1}{\beta'\mu} q_1^* - \frac{\mathrm{i}}{\beta'} q_{1,\xi}^* & 0 & 0 \\ \frac{1}{\beta'^2\mu}\Delta_2^* - \frac{1}{\beta'\mu} q_2^* - \frac{\mathrm{i}}{\beta'} q_{2,\xi}^* & 0 & 0 \end{pmatrix}, \tag{13}$$

where $\Phi = (\phi_1, \phi_2, \phi_3)^T$ is the vector eigenfunction and λ' is the spectral parameter.

From the knowledge of the gauge transformation [38], it is easy to find that system (5) is equivalent to system (10) with the transformation

$$\Phi = \begin{pmatrix} C\lambda & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & C \end{pmatrix}\Psi, \quad \lambda' = \lambda^2, \tag{14}$$

where C is an arbitrary nonzero constant. The Lax pair (5) together with equations (6)–(9) or Lax pair (10) together with equations (11)–(13) not only establishes a scheme for solving the initial-value problem of system (4) but also provides a basis of studying many other integrable properties such as the Hamiltonian structures, conservation laws, symmetry classes and Darboux transformation [36].

3. An infinite number of conservation laws

In this section, starting from the Lax pair (10) together with equations (11)–(13), we will prove the existence of infinitely many independent conservation laws as a further support of the integrability for system (4).

Introducing two new variables

$$\omega_1 = \frac{\phi_2}{\phi_1}, \quad \omega_2 = \frac{\phi_3}{\phi_1}, \tag{15}$$

and taking derivative of ω_j $(j = 1, 2)$ with respect to ξ by use of equation (10a) give rise to the following two Riccati-type equations:

$$\omega_{1,\xi} = -\lambda' q_1 \omega_1^2 + 2\,\mathrm{i}\lambda'\omega_1 - \lambda' q_2\,\omega_1\omega_2 + \frac{1}{\beta'\mu} q_1^*, \tag{16}$$

$$\omega_{2,\xi} = -\lambda' q_2 \omega_2^2 + 2\,\mathrm{i}\lambda'\omega_2 - \lambda' q_1\omega_2\omega_1 + \frac{1}{\beta'\mu} q_2^*. \tag{17}$$

Multiplying equations (16) and (17), respectively, by q_1 and q_2, and expanding $q_1\omega_1$ and $q_2\omega_2$ in power series of $1/\lambda'$ [25]

$$q_1\omega_1 = \sum_{m=0}^{\infty} \lambda'^{-(m+1)}\omega_{1m}(\xi,\tau), \quad q_2\omega_2 = \sum_{m=0}^{\infty} \lambda'^{-(m+1)}\omega_{2m}(\xi,\tau), \tag{18}$$

the recursion formulas for ω_{1m} and ω_{2m} $(m = 0, 1, 2, \ldots)$ can be determined as

$$\omega_{10} = \frac{\mathrm{i}}{2\,\beta'\mu}|q_1|^2, \quad \omega_{20} = \frac{\mathrm{i}}{2\,\beta'\mu}|q_2|^2, \tag{19}$$

$$\omega_{11} = -\frac{\mathrm{i}}{2}\omega_{10}^2 - \frac{\mathrm{i}}{2}\omega_{10}\omega_{20} + \mathrm{i}\frac{q_{1,\xi}}{2\,q_1}\,\omega_{10} - \frac{\mathrm{i}}{2}\omega_{10,\xi}, \tag{20}$$

$$\omega_{21} = -\frac{\mathrm{i}}{2}\omega_{20}^2 - \frac{\mathrm{i}}{2}\omega_{20}\omega_{10} + \mathrm{i}\frac{q_{2,\xi}}{2\,q_2}\,\omega_{20} - \frac{\mathrm{i}}{2}\omega_{20,\xi}, \tag{21}$$

$$\omega_{1m} = -\frac{\mathrm{i}}{2}\left(\sum_{k=0}^{m-1}\omega_{1m-k-1}\omega_{1k} + \sum_{k=0}^{m-1}\omega_{1k}\omega_{2m-k-1} - \frac{q_{1,\xi}}{q_1}\omega_{1m-1} + \omega_{1m-1,\xi}\right) \quad (m > 1), \tag{22}$$

$$\omega_{2m} = -\frac{\mathrm{i}}{2}\left(\sum_{k=0}^{m-1}\omega_{2m-k-1}\omega_{2k} + \sum_{k=0}^{m-1}\omega_{2k}\omega_{1m-k-1} - \frac{q_{2,\xi}}{q_2}\omega_{2m-1} + \omega_{2m-1,\xi}\right) \quad (m > 1). \tag{23}$$

Use of the compatibility condition $(\log\phi_1)_{\xi\tau} = (\log\phi_1)_{\tau\xi}$ yields the following equation in the form of a conservation law:

$$\begin{aligned}[\lambda'(q_1\omega_1 + q_2\omega_2)]_\tau = \Big[&\lambda'(2\,\lambda'\mu - 1)\,(q_1\omega_1 + q_2\omega_2) + \mathrm{i}\,\mu\lambda'\big(\omega_1 q_{1,\xi} + \omega_2 q_{2,\xi}\big)\\ &+ \frac{\lambda'}{\beta'}\,(q_1\omega_1 + q_2\omega_2 - \mathrm{i})\,\big(|q_1|^2 + |q_2|^2\big)\Big]_\xi.\end{aligned} \tag{24}$$

By substituting equations (18) into equation (24) and equating the terms with the same power of $1/\lambda'$, after symbolic manipulations, we can naturally gain a sufficiently large number of conservation laws: $\mathrm{i}\,\frac{\partial\rho_k}{\partial\tau} = \frac{\partial J_k}{\partial\xi}$ $(k = 1, 2, \ldots)$, where ρ_k and J_k $(k = 1, 2, \ldots)$ are the conserved densities and associated fluxes, respectively.

4. The Hirota bilinear method and CDNLS soliton dynamics

The Hirota bilinear method has been used in the construction of the analytic solutions for a number of NLEEs, including difference-differential and integro-differential equations [39]. In essence, this method requires a clever change of dependent variables, a novel differential operator and a perturbation expansion to solve the resulting bilinear equations [40]. Once a given NLEE is bilinearized through certain dependent variable transformation, one can truncate the formal perturbation expansion at different levels and algorithmically obtain a series of solutions, especially multi-soliton solutions. Due to the involvement of a large amount of algebra and calculus calculations which are unmanageable by hand, some computer programs (e.g. HIROTA SINGLE.MAX [40]) have been developed to facilitate the implementation of the Hirota bilinear method. With the aid of symbolic computation which is becoming an important assistant tool for treating soliton equations [40, 41], in the following part we will employ the Hirota bilinear method [39, 40] to construct the analytic one- and two-soliton solutions of system (4).

4.1. Bilinearization

For the convenient purpose of computation and analysis, we first scale out the coefficients β' and μ, and vanish the terms $q_{j,\xi}$ $(j = 1, 2)$ of system (4). Introducing the following scaling:

$$t' = \frac{\tau}{\mu\beta'^2}, \quad x' = \frac{\tau - \xi}{\mu\beta'}, \quad q_j(\xi, \tau) = q_j(x', t') \quad (j = 1, 2), \tag{25}$$

we can write system (4) as the dimensionless CDNLS system [37]

$$\mathrm{i}\, q_{1,t'} + \mathrm{i}\big[q_1\big(|q_1|^2 + |q_2|^2\big)\big]_{x'} + q_{1,x'x'} = 0, \tag{26a}$$
$$\mathrm{i}\, q_{2,t'} + \mathrm{i}\big[q_2\big(|q_2|^2 + |q_1|^2\big)\big]_{x'} + q_{2,x'x'} = 0, \tag{26b}$$

which can be further transformed into the coupled Chen–Lee–Liu equations [42]

$$\mathrm{i}\, u_{1,t'} + \mathrm{i}|u_1|^2 u_{1,x'} + \mathrm{i} u_1 u_2^* u_{2,x'} + u_{1,x'x'} = 0, \tag{27a}$$
$$\mathrm{i}\, u_{2,t'} + \mathrm{i}|u_2|^2 u_{2,x'} + \mathrm{i} u_2 u_1^* u_{1,x'} + u_{2,x'x'} = 0, \tag{27b}$$

with

$$u_j = q_j\, \mathrm{e}^{\frac{\mathrm{i}}{2}\int(|q_1|^2+|q_2|^2)\mathrm{d}x'} \quad (j = 1, 2). \tag{28}$$

Utilizing the rational dependent variable transformations

$$u_1 = \frac{g}{f}, \quad u_2 = \frac{h}{f}, \tag{29}$$

with g, h and f being the complex functions [24], the bilinear representation of system (26) is obtained as

$$\big(\mathrm{i}\, D_{t'} + D_{x'}^2\big)\,(g \cdot f) = 0, \quad \big(\mathrm{i}\, D_{t'} + D_{x'}^2\big)\,(h \cdot f) = 0, \tag{30}$$

$$D_{x'}^2(f \cdot f^*) = \frac{\mathrm{i}}{2}\, D_{x'}(g \cdot g^* + h \cdot h^*), \tag{31}$$

$$D_{x'}(f \cdot f^*) = \frac{\mathrm{i}}{2}\,(|g|^2 + |h|^2), \tag{32}$$

where $D_{t'}$, $D_{x'}$ and $D_{x'}^2$ are all bilinear operators [39] defined by

$$D_{x'}^m D_{t'}^n(a \cdot b) \equiv \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''}\right)^m \left(\frac{\partial}{\partial t'} - \frac{\partial}{\partial t''}\right)^n a(x', t')\, b(x'', t'')\bigg|_{x'=x'',\, t'=t''}. \tag{33}$$

Thus, the soliton solutions of system (26) can be obtained by solving the bilinear equations (30)–(32) with the power series expansion of g, h and f:

$$g = \varepsilon g_1 + \varepsilon^3 g_3 + \varepsilon^5 g_5 + \cdots, \qquad h = \varepsilon h_1 + \varepsilon^3 h_3 + \varepsilon^5 h_5 + \cdots, \qquad f = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 + \cdots, \tag{34}$$

where ε is the formal expansion parameter.

4.2. One-soliton solutions

Aiming to investigate the propagation characteristics of nonlinear Alfvén waves in uniform media, we first derive the hyperbolic one-soliton solutions of system (26). With substitution of the truncations

$$g = \varepsilon g_1, \quad h = \varepsilon h_1, \quad f = 1 + \varepsilon^2 f_2, \tag{35}$$

into equations (30)–(32), the functions g_1, h_1 and f_2 can be solved as

$$g_1 = \alpha^{(1)} e^{\theta^{(1)}}, \quad h_1 = \beta^{(1)} e^{\theta^{(1)}}, \quad f_2 = \gamma e^{\theta^{(1)}+\theta^{(1)*}}, \tag{36}$$

with

$$\theta^{(1)} = k^{(1)} x' + i k^{(1)2} t', \quad \gamma = \frac{i k^{(1)} \left(|\alpha^{(1)}|^2 + |\beta^{(1)}|^2\right)}{2\left(k^{(1)} + k^{(1)*}\right)^2}, \tag{37}$$

where $k^{(1)} + k^{(1)*} \neq 0$, and $\alpha^{(1)}$ and $\beta^{(1)}$ are arbitrary nonzero complex constants. By substituting equations (36) and (37) back into equation (29) with $\varepsilon = 1$ and using equations (25) and (28), the solutions q_j $(j = 1, 2)$ are led to

$$q_1 = \frac{|\alpha^{(1)}| \, e^{i\varphi^{(1)}}}{\sqrt{2|\gamma| \cosh\left(\theta^{(1)} + \theta^{(1)*} + \log|\gamma|\right) + \gamma + \gamma^*}}, \tag{38}$$

$$q_2 = \frac{|\beta^{(1)}| \, e^{i\chi^{(1)}}}{\sqrt{2|\gamma| \cosh\left(\theta^{(1)} + \theta^{(1)*} + \log|\gamma|\right) + \gamma + \gamma^*}}, \tag{39}$$

with

$$\varphi^{(1)} = -\frac{i}{2} \log\left[\frac{\alpha^{(1)} e^{\theta^{(1)}-\theta^{(1)*}} \left(1 + \gamma^* e^{\theta^{(1)}+\theta^{(1)*}}\right)^3}{\alpha^{(1)*}\left(1 + \gamma e^{\theta^{(1)}+\theta^{(1)*}}\right)^3}\right],$$

$$\chi^{(1)} = -\frac{i}{2} \log\left[\frac{\beta^{(1)} e^{\theta^{(1)}-\theta^{(1)*}} \left(1 + \gamma^* e^{\theta^{(1)}+\theta^{(1)*}}\right)^3}{\beta^{(1)*}\left(1 + \gamma e^{\theta^{(1)}+\theta^{(1)*}}\right)^3}\right],$$

where q_1 and q_2, respectively, correspond to the right- and left-polarized components of the Alfvén soliton.

It is obvious that $|\gamma + \gamma^*| < 2|\gamma|$ when $k^{(1)} + k^{(1)*} \neq 0$, so the right- and left-polarized soliton envelopes exhibit no singularity and maintain stable bell profiles during the propagation. The amplitudes of q_j $(j = 1, 2)$ are respectively given by

$$A_1 = \frac{|\alpha^{(1)}|}{\sqrt{2|\gamma| + \gamma + \gamma^*}}, \quad A_2 = \frac{|\beta^{(1)}|}{\sqrt{2|\gamma| + \gamma + \gamma^*}}, \tag{40}$$

which are both reached at the center of mass of the soliton:

$$\xi_c(\tau) = \frac{(4\beta + i k^{(1)} - i k^{(1)*} - 4)\tau}{\beta'} + \frac{\mu\beta' \log|\gamma|}{k^{(1)} + k^{(1)*}}. \tag{41}$$

By using the vanishing boundary conditions $q_j|_{x'\to\pm\infty} = 0$ $(j = 1, 2)$, we can compute the corresponding conserved total energy of two components of the single Alfvén soliton:

$$E = \int_{-\infty}^{+\infty} \left(|q_1|^2 + |q_2|^2\right) dx' = 8 \tan^{-1}\left(\frac{2|k^{(1)}| - i k^{(1)} + i k^{(1)*}}{|k^{(1)} + k^{(1)*}|}\right), \tag{42}$$

which suggests that E is only dependent on the parameter $k^{(1)}$.

Finite-amplitude Alfvén waves have been observed in a variety of plasmas with different values of β [4, 43]. For example, in the solar wind the range of β is taken as $0 \leqslant \beta < 1$ and $\beta > 1$. To understand how the parameter β acts on the propagation of Alfvén solitons, a simple way is to make an analysis of the soliton width and velocity

$$W_\beta = \frac{|\mu\beta'|}{|k^{(1)} + k^{(1)*}|}, \quad V_\beta = 1 + \frac{i\left(k^{(1)} - k^{(1)*}\right)}{\beta'}. \tag{43}$$

The sensitive dependence of the soliton width and velocity on β takes place in the neighborhood of $\beta = 1$, at which the soliton width $W_\beta \to 0$ and the soliton velocity $V_\beta \to \infty$. The propagation direction of the Alfvén soliton always keeps positive when $0 \leqslant \beta < 1$, while for $\beta > 1$ there must be a turning point where the direction changes from negative to positive. If β is large enough, the soliton width $W_\beta \to \infty$ and the soliton velocity $V_\beta \to 1$.

4.3. Two-soliton solutions

The Alfvén soliton interaction has been an important subject of considerable experimental studies, e.g. the experiments conducted with the Large Plasma Device [44]. This triggers our motivation to construct the two-soliton solutions of system (4), which are expected to be very helpful for the theoretical explanation of interaction phenomena between nonlinear Alfvén waves. We substitute

$$g = \varepsilon g_1 + \varepsilon^3 g_3, \quad h = \varepsilon h_1 + \varepsilon^3 h_3, \quad f = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4, \tag{44}$$

into equations (30)–(32) and solve the resulting equations, yielding

$$g_1 = \alpha^{(1)} \mathrm{e}^{\theta^{(1)}} + \alpha^{(2)} \mathrm{e}^{\theta^{(2)}}, \quad h_1 = \beta^{(1)} \mathrm{e}^{\theta^{(1)}} + \beta^{(2)} \mathrm{e}^{\theta^{(2)}}, \tag{45}$$

$$g_3 = \delta^{(1)} \mathrm{e}^{\theta^{(1)}+\theta^{(2)}+\theta^{(2)*}} + \delta^{(2)} \mathrm{e}^{\theta^{(1)}+\theta^{(2)}+\theta^{(1)*}}, \quad h_3 = \nu^{(1)} \mathrm{e}^{\theta^{(1)}+\theta^{(2)}+\theta^{(2)*}} + \nu^{(2)} \mathrm{e}^{\theta^{(1)}+\theta^{(2)}+\theta^{(1)*}}, \tag{46}$$

$$f_2 = \gamma \mathrm{e}^{\theta^{(1)}+\theta^{(1)*}} + \zeta \mathrm{e}^{\theta^{(1)}+\theta^{(2)*}} + \kappa \mathrm{e}^{\theta^{(2)}+\theta^{(1)*}} + \pi \mathrm{e}^{\theta^{(2)}+\theta^{(2)*}}, \quad f_4 = \sigma \mathrm{e}^{\theta^{(1)}+\theta^{(2)}+\theta^{(1)*}+\theta^{(2)*}}, \tag{47}$$

where

$$\theta^{(1)} = k^{(1)} x' + \mathrm{i} k^{(1)^2} t', \quad \theta^{(2)} = k^{(2)} x' + \mathrm{i} k^{(2)^2} t',$$

$$\delta^{(1)} = \frac{\alpha^{(1)} \pi \left(k^{(1)} - k^{(2)}\right)}{k^{(1)} + k^{(2)*}} - \frac{\alpha^{(2)} \zeta \left(k^{(1)} - k^{(2)}\right)}{k^{(2)} + k^{(2)*}},$$
$$\delta^{(2)} = \frac{\alpha^{(1)} \kappa \left(k^{(1)} - k^{(2)}\right)}{k^{(1)} + k^{(1)*}} - \frac{\alpha^{(2)} \gamma \left(k^{(1)} - k^{(2)}\right)}{k^{(1)*} + k^{(2)}},$$

$$\nu^{(1)} = \frac{\beta^{(1)} \pi \left(k^{(1)} - k^{(2)}\right)}{k^{(1)} + k^{(2)*}} - \frac{\beta^{(2)} \zeta \left(k^{(1)} - k^{(2)}\right)}{k^{(2)} + k^{(2)*}},$$
$$\nu^{(2)} = \frac{\beta^{(1)} \kappa \left(k^{(1)} - k^{(2)}\right)}{k^{(1)} + k^{(1)*}} - \frac{\beta^{(2)} \gamma \left(k^{(1)} - k^{(2)}\right)}{k^{(1)*} + k^{(2)}},$$

$$\gamma = \frac{\mathrm{i} k^{(1)} \left(|\alpha^{(1)}|^2 + |\beta^{(1)}|^2\right)}{2 \left(k^{(1)} + k^{(1)*}\right)^2}, \quad \zeta = \frac{\mathrm{i} k^{(1)} \left(\alpha^{(1)} \alpha^{(2)*} + \beta^{(1)} \beta^{(2)*}\right)}{2 \left(k^{(1)} + k^{(2)*}\right)^2},$$
$$\kappa = \frac{\mathrm{i} k^{(2)} \left(\alpha^{(1)*} \alpha^{(2)} + \beta^{(1)*} \beta^{(2)}\right)}{2 \left(k^{(1)*} + k^{(2)}\right)^2},$$

$$\pi = \frac{\mathrm{i} k^{(2)} \left(|\alpha^{(2)}|^2 + |\beta^{(2)}|^2\right)}{2 \left(k^{(2)} + k^{(2)*}\right)^2}, \quad \sigma = \frac{\gamma \pi |k^{(1)} - k^{(2)}|^2}{\left(k^{(2)} + k^{(1)*}\right)\left(k^{(1)} + k^{(2)*}\right)} - \frac{\zeta \kappa |k^{(1)} - k^{(2)}|^2}{\left(k^{(1)} + k^{(1)*}\right)\left(k^{(2)} + k^{(2)*}\right)},$$

with $k^{(1)} + k^{(1)*} \neq 0$, $k^{(2)} + k^{(2)*} \neq 0$, $k^{(1)} + k^{(2)*} \neq 0$ and $\alpha^{(1)}$, $\alpha^{(2)}$, $\beta^{(1)}$ and $\beta^{(2)}$ as arbitrary nonzero complex constants. By taking $\varepsilon = 1$, the two-soliton solutions for system (26) are obtained as

$$q_1 = \frac{(1 + f_2^* + f_4^*)(g_1 + g_3)}{(1 + f_2 + f_4)^2}, \quad q_2 = \frac{(1 + f_2^* + f_4^*)(h_1 + h_3)}{(1 + f_2 + f_4)^2}, \tag{48}$$

where f_2, f_4, g_1, g_3, h_1 and h_3 have been obtained as above.

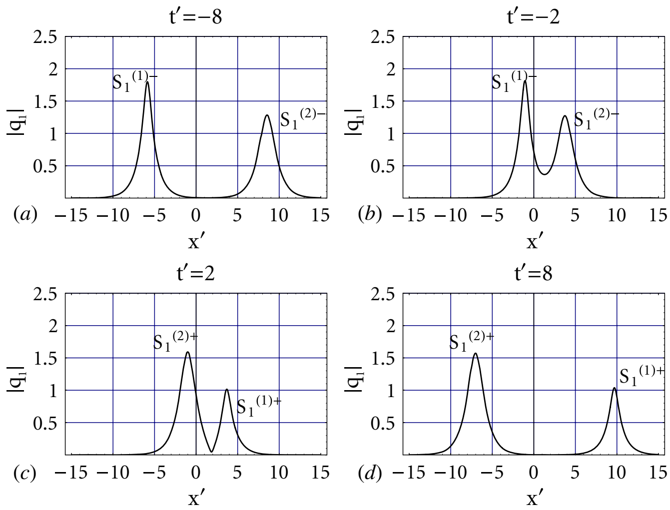

Figure 1. Amplitude-changing collision of the right-polarized Alfvén soliton components $S_1^{(1)}$ and $S_1^{(2)}$ with $k^{(1)} = 1 + 0.5\,\mathrm{i}$, $k^{(2)} = 1 - 0.5\,\mathrm{i}$, $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = -1$, $\beta^{(2)} = 0$.

4.4. *Interaction properties of Alfvén solitons with right and left polarizations*

To obtain a clearer understanding of the collision mechanism of Alfvén solitons, we perform an asymptotic analysis of solutions (48) in a manner like in [33–35]. Without loss of generality, we assume that $k_I^{(1)}k_I^{(2)} < 0$ and $k_R^{(1)}k_R^{(2)} > 0$ ($k_I^{(l)} = \mathrm{i}(k^{(l)*} - k^{(l)})/2$ and $k_R^{(l)} = (k^{(l)} + k^{(l)*})/2$ for $l = 1, 2$), which corresponds to the head-on soliton collision. It can be found that the Alfvén solitons with right- and left-polarized components possess the following interaction properties.

1. *The velocities and widths of Alfvén solitons keep invariant before and after the collision.* In the reference frame x'–t', the Alfvén soliton velocity uniquely depends on the parameter $k_I^{(l)}$, while the width on $k_R^{(l)}$.
2. *The amplitude-changing collision occurs for two components of Alfvén solitons under certain parametric condition.* Through simple calculations, we find that each component of the Alfvén solitons admits the amplitude-preserving collision under the following condition:

$$\begin{aligned}&\mathrm{Re}\Big[\alpha^{(2)*}\beta^{(2)*}\left(|\alpha^{(1)}|^2 + |\beta^{(1)}|^2\right)\left(\alpha^{(1)}\beta^{(2)} - \alpha^{(2)}\beta^{(1)}\right)\left(\alpha^{(2)}\alpha^{(1)*} + \beta^{(2)}\beta^{(1)*}\right)|k^{(1)}|^2 k^{(2)}\\&+ \alpha^{(1)*}\beta^{(1)*}\left(|\alpha^{(2)}|^2 + |\beta^{(2)}|^2\right)\left(\alpha^{(2)}\beta^{(1)} - \alpha^{(1)}\beta^{(2)}\right)\left(\alpha^{(1)}\alpha^{(2)*} + \beta^{(1)}\beta^{(2)*}\right)\left|k^{(2)}\right|^2 k^{(1)}\Big]\\&\quad = 0.\end{aligned} \tag{49}$$

When the relevant parameters do not satisfy condition (49), solutions (48) describe the amplitude-changing collision between right- and left-polarized components of two Alfvén solitons. In the illustration, figures 1(*a*)–(*d*) show that the amplitudes of two right-polarized Alfvén soliton components $S_1^{(1)}$ and $S_1^{(2)}$, respectively, get compressed and enhanced after the collision. With an appropriate choice of parameters, it is also possible that the amplitude-vanishing collision takes place, namely, the amplitude for one of two colliding Alfvén soliton components totally becomes zero after the collision, as illustrated

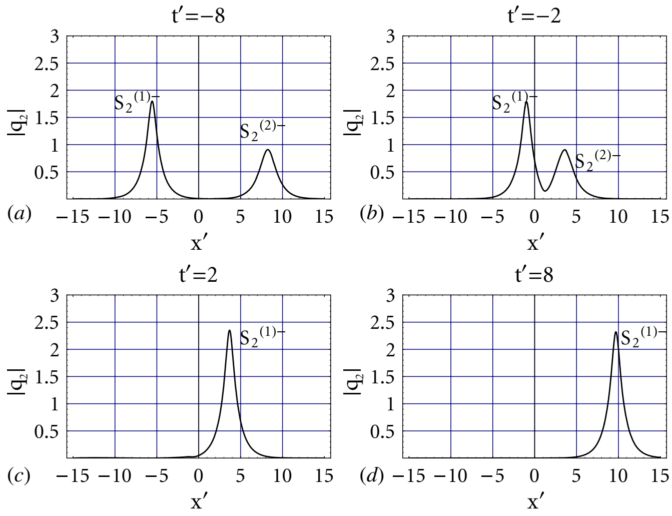

Figure 2. Amplitude-vanishing collision of the left-polarized Alfvén soliton components $S_2^{(1)}$ and $S_2^{(2)}$ with the same parameters as those in figure 1.

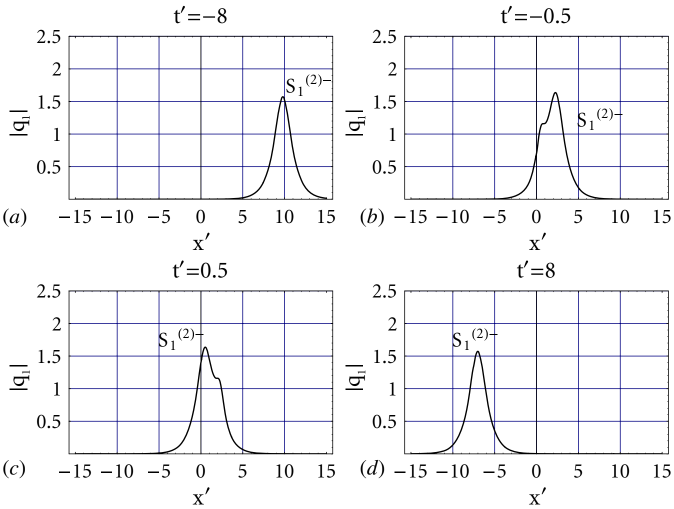

Figure 3. Evolution of the colliding Alfvén soliton $\mathbf{S}^{(2)}$ only with the right-polarized component. The related parameters are chosen as $k^{(1)} = 1 + 0.5\,\mathrm{i}$, $k^{(2)} = 1 - 0.5\,\mathrm{i}$, $\alpha^{(1)} = 0$, $\alpha^{(2)} = 1$, $\beta^{(1)} = -1$, $\beta^{(2)} = 0$.

in figures 2(*a*)–(*d*). In particular, if $\alpha^{(1)} = \beta^{(2)} = 0$, the soliton $\mathbf{S}^{(1)}$ just comprises the left-polarized component $S_2^{(1)}$ and the soliton $\mathbf{S}^{(2)}$ only contains the right-polarized component $S_1^{(2)}$. In this case, one can observe from figures 3(*a*)–(*d*) and figures 4(*a*)–(*d*) that $S_2^{(1)}$ and $S_1^{(2)}$ with different $\omega^{(1)} = {k^{(1)}}^2$ and $\omega^{(2)} = {k^{(2)}}^2$ just undergo a slight shape change in the neighborhood of $t = 0$.

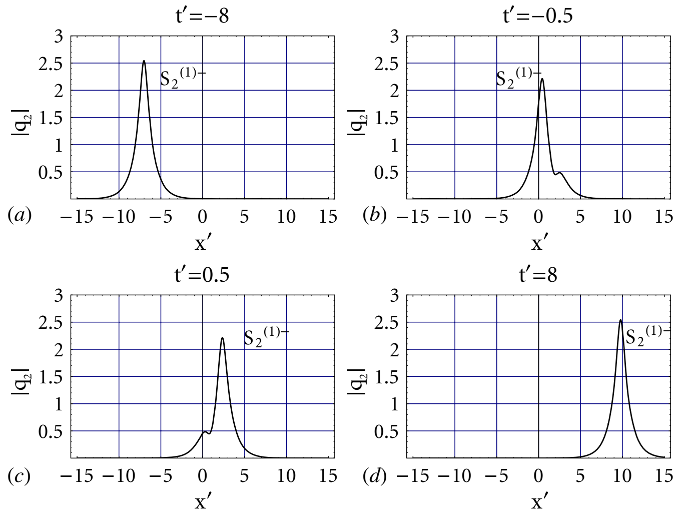

Figure 4. Evolution of the colliding Alfvén soliton $\mathbf{S}^{(1)}$ only with the left-polarized component. The choice of parameters follows figure 3.

3. *The total energy of right- and left-polarized components for each colliding Alfvén soliton is conserved during the collision.* By verification, it is not difficult to find that

$$E_1^{(l)-} + E_2^{(l)-} = E_1^{(l)+} + E_2^{(l)+} = 8\tan^{-1}\left(\frac{2\,|k^{(l)}| - \mathrm{i}\,k^{(l)} + \mathrm{i}\,k^{(l)*}}{|k^{(l)} + k^{(l)*}|}\right), \qquad (50)$$

where $E_1^{(l)-}$ and $E_2^{(l)-}$ represent the energies of the right- and left-polarized components of the lth Alfvén soliton before the collision, and $E_1^{(l)+}$ and $E_2^{(l)+}$ represent the energies of the right- and left-polarized components of the lth Alfvén soliton after the collision.

It is noted that we can further derive the N-soliton solutions ($N \geqslant 3$) of system (26) and analyze the collision dynamics among three or more Alfvén solitons in the same way like the two-soliton case.

5. Conclusions

In the present paper, the DNLS equation has been generalized into the CDNLS system, i.e. system (4), for describing nonlinear Alfvén waves with right and left polarizations in space and laboratory plasmas. Trying to show the integrability of the CDNLS system, we have constructed two types of Lax pairs which are gauge-equivalent to each other, and further proved the existence of infinitely many independent conservation laws. Another concern of this paper is to reveal underlying unusual Alfvén soliton behaviors resulting from the derivative cubic nonlinearity coupling. Via the Hirota bilinear method and assisted by symbolic computation, we have obtained the analytic one- and two-soliton solutions of system (26). On this basis, we have discussed the influence of the parameter β on the propagation of Alfvén solitons. The qualitative analysis has shown that the CDNLS soliton interaction is characterized by the invariant soliton velocities and widths, changeable soliton amplitudes under certain parametric condition and conserved total energy of two components for each colliding soliton. The parametric condition of the amplitude-preserving collision occurring in each component of

Alfvén solitons has been explicitly presented. Finally, we remark that all the analytical results about system (26) might be applicable to some realistic plasma situations if those dimensional units are included via transformations (3) and (25).

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